Analytical Derivation of a Two-Section Impedance Transformer for a Frequency and Its First Harmonic

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***Abstract*—The feasibility of an electrically small two-section transformer (total length one-third-wavelength at the fundamental) capable of achieving ideal impedance matching at a fundamental frequency and its first harmonic is demonstrated analytically. To achieve this, the exact solution to the resulting transcendental transmission line equations for two sections is obtained with no restrictions. The parameters of the transformer are presented in explicit closed form and are exact. The results of this study are useful for a number of practical design problems, including dual-band antennas and RF circuits in general.**

***Index Terms*—Dual frequency, impedance matching, transformer, transmission lines.**

I. Introduction

ONE OF the major drawbacks of the quarter-wave transformers is the narrow bandwidth. Though a large number of alternative schemes have been developed to improve the bandwidth [1]–[3], it appears that little has been done to address the case of operation at the first harmonic of the fundamental. The current trend is toward smaller and more efficient RF front ends in commercial and military systems, making this problem of significant interest.

A new line impedance transformer has been introduced [4]. It is a dual band two-section one-third wavelength transformer length that operates at the fundamental frequency f_1 and its first harmonic $2f_1$. The transformer was designed by numerical solution of the transcendental equations obtained by enforcing operation at the fundamental frequency and its first harmonic. In addition, a design equation was presented [4, eq. (2)], which, although not analytically exact, was numerically *nearly exact*. It was further stated in [4] that Mathematica [5] was used to prove that the two-section transformer was not exact, but for engineering applications "effectively exact" for impedance transform ratios K not sufficiently high.

Here, we present an alternative treatment of the two-section transformer analysis of [4], and obtain by elementary means an exact analytical solution to the resulting transcendental equations, leading to practical design equations, which are validated numerically. Here, we demonstrate that the two-section transformer provides a true exact solution to the present dual-frequency problem, under all impedance transform ratios.

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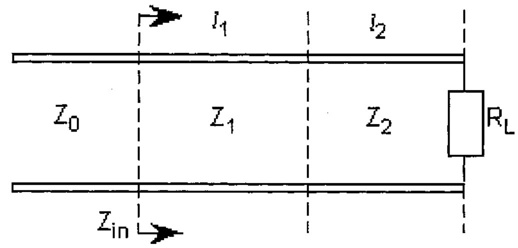

Fig. 1. Two-section dual band transformer.

II. Solution

The input impedance Z_{in} of the two-section line, shown in Fig. 1, is given by

$$Z_{in} = Z_1 \frac{Z_L' + jZ_1 \tan(\beta\ell_1)}{Z_1 + jZ_L' \tan(\beta\ell_1)} \tag{1}$$

$$Z_L' = Z_2 \frac{R_L + jZ_2 \tan(\beta\ell_2)}{Z_2 + jR_L \tan(\beta\ell_2)} \tag{2}$$

where ℓ_1 and ℓ_2 are the line lengths, β is the wavenumber, and R_L the load impedance. Setting Z_{in} to the desired value, Z_0 establishes the impedance match transcendental equations. All parameters are illustrated in Fig. 1.

By separating the real and imaginary components of the above equations, and by applying the resulting expressions to frequencies f_1 and $2f_1$ we obtain four equations. If we assume all parameters real, it is possible to solve the four transcendental equations for the four unknowns (the two line lengths and the wave impedances Z_1 and Z_2) by standard algebraic means.

The solution for the smallest two-section transformer we can get is given by the following line lengths:

$$\ell_1 = \ell_2 \qquad \ell_2 = \frac{\pi}{3\beta_1}. \tag{3}$$

For K, the dimensionless impedance transform ratio parameter, defined as in [4], is

$$K = Z_0/R_L. \tag{4}$$

The associated line impedances become

$$\frac{Z_1}{R_L} = \sqrt{\frac{K}{6}(1-K) + \sqrt{\left[\frac{K}{6}(1-K)\right]^2 + K^3}} \tag{5a}$$

$$Z_2 = Z_0 R_L / Z_1 \tag{5b}$$

where the last identity is essentially the so-called antimetry condition [6].

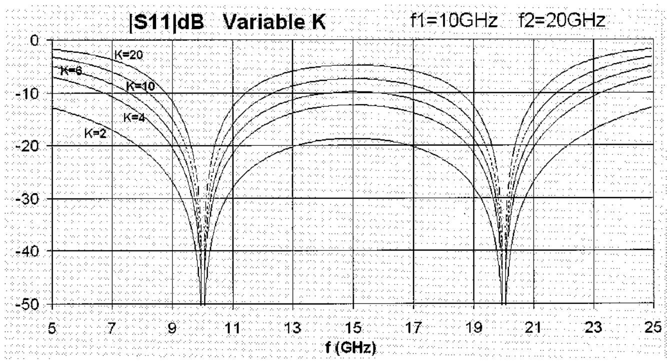

Fig. 2. Case of a fundamental $f_1 = 10$ GHz and its first harmonic for variable impedance transform ratios K. The quality of the solution is evident from the figure.

The solution is elementary in nature, explicit, and in closed form. In view of (3), the total length of the transformer is given by, $\ell_1 + \ell_2 = \lambda_1/3$, i.e., one-third-wavelengths calculated at the fundamental frequency f_1. This point is in congruence with the approximate solution of [4].

A symmetry property can be obtained by purely algebraic manipulations of (4) and (5). It can be shown that the solution for $K = K_0$ is related to the solution for $1/K$ via

$$\left.\frac{Z_1}{R_L}\right|_{K=K_0} = \left.\frac{Z_2}{Z_0}\right|_{K=1/K_0} \qquad \left.\frac{Z_2}{R_L}\right|_{K=K_0} = \left.\frac{Z_1}{Z_0}\right|_{K=1/K_0}. \tag{6}$$

III. Numerical Proof of the Exact Solution

The simulation corresponds to a fundamental $f_1 = 10$ GHz and illustrates the fact that the present solution with two sections is exact. Return loss S_{11} data as a function of the frequency is presented in Fig. 2 for values of K ranging from 2 to 20. Fig. 3 shows the $K = 3$ case in a Smith's chart format to illustrate the loop response. In all cases, the frequency runs from 5 to 25 GHz.

We also considered the validation of the properties (6) corresponding to K and its inverse $1/K$. Space does not allow for inclusion of further results; suffice it to say that simulations were performed to verify (6), and its validity has been demonstrated.

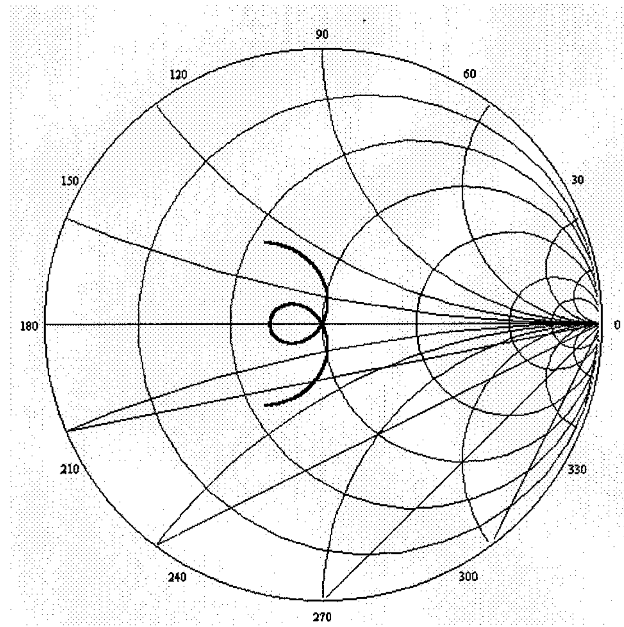

Fig. 3. Smith's Chart to illustrate the nature of the loop response. Impedance match at 10 and 20 GHz. $K = 3$.

IV. Conclusion

A novel and elementary two-section impedance transformer of one-third wavelength total length has been shown capable of operation at the first harmonic of the fundamental under unrestricted impedance transforming conditions. Exact closed-form design equations are presented, and the results are validated numerically even for large impedance transform ratios K. This work improves on the recent work reported in [4].

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